# 理想流体力学演習問題

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1.(25) 二次元流れの速度成分が $u = x - 4y, v = -4x - y$ で与えられる流れは (1) 理論上存在するか。(2) 流れの関数を求めよ。(3) もし渦無し流れであればその速度ポテンシャルを求めよ。2.(25) 二次元ポテンシャル流れにおいて、その速度成分が、$u = ax + by, v = cx + dy$ で与えられるとき、(1) 定数 $a, b, c, d$ の関係、(2) 速度ポテンシャルおよび流れの関数を求めよ。3.(25) 連続の式を満足するための次の速度成分を求めよ。

$$
\begin{aligned}
&(1)u = x^2 + y^2 + z^2, v = -xy - yz - xz, w = \\
&(2)u = ln(y^2 + az^2), v = sin(x^2 + y^2), w =
\end{aligned}
$$

4.(25) 二次元の渦流れにおいて，速度成分が $u = x + y,\ v = x^2 - y$ なる流れは理論上存在しうるか．(2) その流れの流線を求めよ．(3) 直線 $x = \pm 1, y = \pm 1$ で区切られた正方形のまわりの循環値を求めよ

（解）

1.

$$
\begin{aligned}
&(1)divV = 1 - 1 = 0 \\
&(2)u = \frac{\partial \psi}{\partial y} = x - 4y; \psi = xy - 2y^2 + f(x) \\
&-v = \frac{\partial \psi}{\partial x} = 4x + y; \psi = 2x^2 + xy + f(y) \\
&\psi = xy - 2y^2 + 2x^2 + c \\
&\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -4 - (-4) = 0 \\
&(3)u = \frac{\partial \phi}{\partial x} = x - 4y; \phi = 1/2x^2 - 4xy + f(y) \\
&v = \frac{\partial \phi}{\partial y} = -x - y; \phi = -4xy - 1/2y^2 + f(x) \\
&\phi = 1/2x^2 - 1/2y^2 - 4xy; c
\end{aligned}
$$

2.

$$
\begin{aligned}
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0 \\
&u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy \\
&\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y) \\
&\psi = axy + \frac{1}{2}(by^2 - cx^2) + const. \\
&\text{For irrotational flow, } \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x},\ b = c,\ \psi = axy + \frac{b}{2}(y^2 - x^2) + const. \\
&(3)u = \frac{\partial \phi}{\partial x} = ax + by; \phi = 1/2ax^2 + bxy + f(y)
\end{aligned}
$$

$$v = \frac{\partial \phi}{\partial y} = cx + dy; \phi = 1/2dy^2 + cxy + f(x)$$
$$\phi = 1/2a(x^2 - y^2) + bxy + c$$

3.

$$(1)\frac{\partial u}{\partial x} = 2x, \frac{\partial v}{\partial y} = -x - z, divV = 0$$
$$\frac{\partial w}{\partial z} = -x + z, w = -xz + 1/2z^2 + f(x, y)$$
$$(2)\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 2ycos(x^2 + y^2)$$
$$w = -2yzcos(x^2 + y^2) + f(x, y)$$

4.

$$(1)\ divV = 0$$
$$(2)u = \frac{\partial \psi}{\partial y} = x + y; \psi = xy + 1/2y^2 + f(x)$$
$$-v = \frac{\partial \psi}{\partial x} = -x^2 + y; \psi = -1/3x^3 + xy + f(y)$$
$$\psi = -1/3x^3 + 1/2y^2 + xy + c; 1/3x^3 - 1/2y^2 - xy = c$$
$$(3)\Gamma = \int_{-1}^{1}\int_{-1}^{1}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})dxdy = \int_{-1}^{1}\int_{-1}^{1}(2x - 1)dxdy = -4$$