理想流体力学 試験問題

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1. (25) 速度成分が $u = ax + by,\ v = cx + dy$ で示される流れが非圧縮性流体となるための条件を示せ．また，流れが渦なし流れとした場合の流れ関数を求めよ．
2. (30) 複素ポテンシャルが次式で表される流れの型を説明し、かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ．

$$(1)\ w = aze^{i\alpha}\ (\alpha > 0),\ (2)\ w = z^n\ (n = \frac{1}{2}),\ (3)\ w = -5i\ln z + 3z,\ (4)\ w = 2z + 3\ln z$$

3. (25) 速度 U の一様流れ中に強さ Q の吹き出しが原点にある場合、この流れ場に作用する力を求めよ．
4. (20) 二次元の渦流れで，その速度成分が $v_r = 0,\ v_\theta = \omega r$ なるときの渦度を求めよ．

（解）

1.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0$$

$$u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy$$

$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$

$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$

For irrotational flow, $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x},\ b = c,\ \psi = axy + \dfrac{b}{2}(y^2 - x^2) + const.$

2.

(1) Parallel flow with $\theta = \alpha$

$$w = ar\{(\cos(\theta + \alpha) + i\sin(\theta + \alpha)\}$$

$$\varphi = ar\cos(\theta + \alpha), \quad \psi = ar\sin(\theta + \alpha)$$

$$\frac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i\sin\alpha) = u - iv$$

$$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$$

(2) Corner flow with $\theta = 2\pi$

$$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$

$$\varphi = r^n\cos n\theta, \quad \psi = r^n\sin n\theta$$

$$For\ n = \frac{1}{2}, \quad \varphi = r^{1/2}\cos\frac{\theta}{2}, \quad \psi = r^{1/2}\sin\frac{\theta}{2}$$

(3) Parallel (U=3)+circulation($\Gamma = 10\pi$) flow

$$w = -5i\ln(re^{i\theta}) + 3re^{i\theta} = -5\ln r + 5\theta + 3r(\cos\theta + i\sin\theta)$$

$$\varphi = 5\theta + 3r\cos\theta, \quad \psi = 3r\sin\theta - 5\ln r$$

(4) Parallel flow(U=2)+source flow($Q = 6\pi$)

$$w = 2re^{i\theta} + 3\ln(re^{i\theta})$$

$$\varphi = 2r\cos\theta + 3\ln r, \quad \psi = 2r\sin\theta + 3\theta$$

3.

$$
\begin{aligned}
&w = Uz + m \ln z, \quad m = \frac{Q}{2\pi} \\
&\frac{dw}{dz} = U + \frac{m}{z} \\
&(\frac{dw}{dz})^2 = U^2 + \frac{m^2}{z^2} + \frac{2Um}{z} \\
&F_x - iF_y = \frac{i\rho}{2} \oint (\frac{dw}{dz})^2 dz = \frac{i\rho}{2} 2Um(2\pi i) \\
&F_x = -\rho UQ, \quad F_y = 0
\end{aligned}
$$

4.

$$
\begin{aligned}
&v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0, \quad \psi = f(r) \\
&v_\theta = -\frac{\partial \psi}{\partial r} = \omega r, \quad \psi = -\frac{1}{2}\omega r^2 + f(\theta) \\
&\psi = -\frac{1}{2}\omega r^2 = -\frac{1}{2}\omega(x^2 + y^2) \\
&\zeta = -\nabla^2 \psi = -(-\omega - \omega) = 2\omega
\end{aligned}
$$