流体力学III試験問題

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1. 次の流れを説明し、これらはすべて理論上存在しうる流であり、かつ (4) 以外はすべてうずなし流れであることを示せ。

$$(1)\ \psi = 15y, \quad (2)\ \psi = 17.3y - 10x, \quad (3)\ \psi = -20x, \quad (4)\ \psi = -5x^2$$

2. 次の関数で速度ポテンシャルの存在するものを示せ.

$$(1)\ F = x + y + z, \quad (2)\ F = x + xy + xyz, \quad (3)\ F = \ln x, \quad (4)\ F = \sin(x + y + z)$$

3. 二次元の軸に平行な流れで速度が $y = 0$ で 0, $y = 4$ で $20m/s$ で直線的に変化しているとき，その流れの関数を求めよ．また流れは渦なし流れか.

4. 流体の速度成分が $u = yz + t,\ v = xz - t,\ w = xy$ で与えられるとき点 (1,1,1) における流体の加速度の成分を t で表せ.

(解)

1.

$$(1)\ u = \frac{\partial \psi}{\partial y} = 15, \quad v = \frac{\partial \psi}{\partial x} = 0, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$(2)\ u = \frac{\partial \psi}{\partial y} = 17.3, \quad v = \frac{\partial \psi}{\partial x} = 10, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$(3)\ u = \frac{\partial \psi}{\partial y} = 0, \quad v = \frac{\partial \psi}{\partial x} = 20, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$(4)\ u = \frac{\partial \psi}{\partial y} = 0, \quad v = \frac{\partial \psi}{\partial x} = 10x, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 10(\text{rotational})$$

2.

$$(1)\ \nabla^2 F = 0, \quad (2)\ \nabla^2 F = 0, \quad (3)\ \nabla^2 = -\frac{1}{x^2}, \quad (4)\ \nabla^2 F = -3\sin(x + y + z)$$

3.

$$u = \frac{\partial \psi}{\partial y} = 5y, \quad \psi = \frac{5}{2}y^2$$

4.

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 1 + vz + wy = 1 + (xz - t)z + xy^2,\ at\ (1,1,1),\ \frac{dw}{dt} = 3 - t$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= -1 + uz + wx = -1 + (yz + t)z + x^2,\ at\ (1,1,1),\ \frac{dw}{dt} = t + 1$$

$$\frac{dw}{dt} = uy + vx = (yz + t)y + (xz - t)x,\ at\ (1,1,1),\ \frac{dw}{dt} = 2$$