

流体力学 II 試験問題 (1)

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1. (25) 次の速度分布に対する排除厚さ, 運動量厚さおよび形状係数を求めよ.

$$(1) \frac{u}{U} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}, \quad (2) \frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

2. (25) 図に示す単位幅のせきを超えて流れる流量 Q を表す式を求めよ. ただし図に示す物理量のほか, 関連するものは重力の加速度 g のみとする.

3. (25) 同じ断面積, 同じ長さを持つ円管と正三角形断面の管を流れる乱流において, 管摩擦損失水頭が等しければ流量比は幾らになるか. ただし, 両管の管摩擦係数は等しいものとする.

4. (25) 直径 25 cm, 長さ 85 m の円管で 3.5 mAq の圧力損失がある場合について次の値を計算せよ: (1) 円管壁におけるせん断応力, (2) 円管の中心より 3 cm の位置におけるせん断応力, (3) 摩擦速度, (4) 摩擦係数を 0.03 としたときの円管内の平均速度.

(解)

1.

$$(1) \delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy = \frac{\delta}{3}, \quad \frac{\delta^*}{\delta} = \frac{1}{3}$$

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy = \delta \int_0^1 (2\eta - \eta^2) - (2\eta - \eta^2)^2 d\eta$$

$$= \delta \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4) = \delta \left(1 - \frac{5}{3} + 1 - \frac{1}{5}\right) = \frac{2}{15}\delta, \quad \frac{\theta}{\delta} = \frac{2}{15}$$

$$H = \frac{\delta^*}{\theta} = \frac{1}{3} \times \frac{15}{2} = 2.5$$

$$(2) \delta^* = \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy = \frac{\delta}{8}, \quad \frac{\delta^*}{\delta} = \frac{1}{8}$$

$$\theta = \delta \int_0^1 (\eta^{1/7} - \eta^{2/7}) = \delta \left(\frac{7}{8} - \frac{7}{9}\right) \frac{7}{72} \delta, \quad \frac{\theta}{\delta} = \frac{7}{72}$$

$$H = \frac{\delta^*}{\theta} = \frac{1}{8} \times \frac{72}{7} = 1.286$$

2.

Q, H, D, g

$n = 4, i = 2, m = n - i = 2$ (n, d, ρ - primary variables)

$$\pi_1 = Q^\alpha H^\beta g^\gamma = L^{2\alpha} T^{-\alpha} L^\beta L^\gamma T^{-2\gamma}$$

$$L: 2\alpha + \beta + \gamma = 0, \quad T: -\alpha - 2\gamma = 0$$

$$\alpha = 1(\text{take}), \quad \beta = -\frac{3}{2}, \quad \gamma = -\frac{1}{2}$$

$$\pi_1 = \frac{Q}{\sqrt{gH^3}}, \quad \pi_2 = Q^\alpha H^\beta D^\gamma = L^{2\alpha} T^{-\alpha} L^\beta L^\gamma$$

$$L: 2\alpha + \beta + \gamma = 0, \quad T: -\alpha = 0$$

$$\beta = -\gamma, \quad \pi_2 = \left(\frac{H}{D}\right)^\beta$$

$$\pi_1 = \varphi(\pi_2) = \varphi\left(\frac{H}{D}\right), \quad Q = \varphi\left(\frac{H}{D}\right) \sqrt{gH^3}$$

3.

$$\begin{aligned}h_1 &= \lambda \frac{l}{d} \frac{v_1^2}{2g}, & h_2 &= \lambda \frac{l}{4m} \frac{v_2^2}{2g}, \\m &= \frac{\sqrt{3}}{12} a, & 4m &= \frac{\sqrt{3}}{3} a, & \frac{\pi d^2}{4} &= \frac{\sqrt{3}}{4} a^2 \\ \frac{a}{d} &= \left(\frac{\pi}{\sqrt{3}}\right)^{1/2} \\ \frac{Q_2}{Q_1} &= \frac{Av_2}{Av_1} = \left(\frac{4m}{d}\right)^{1/2} = \left(\frac{a}{\sqrt{3}d}\right)^{1/2} = \left[\frac{1}{\sqrt{3}d} \left(\frac{\pi}{\sqrt{3}}\right)^{1/2}\right]^{1/2} = 0.882\end{aligned}$$

4.

$$(1) \tau_w \pi d dx = dp A$$

$$\tau_w \pi d = \frac{dp}{dx} \frac{\pi d^2}{4}, \quad \tau_w = \frac{d}{4} \frac{dp}{dx}$$

$$\tau_w = \frac{0.25}{4} \times \frac{3.5 \times 10^3 g}{85} = 25.1 Pa (2.57 \times 10^{-4} kgf/cm^2)$$

$$(2) \frac{\tau_w}{\tau} = \frac{r_o}{r}, \quad \tau = 25.1 \times \frac{3}{12.5} = 6.04 Pa$$

$$(3) v^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{25.1}{10^3}} = 0.158 m/s$$

$$(4) h = \lambda \frac{L}{d} \frac{v^2}{2g}, \quad v = \sqrt{2g \times 3.5 \times 0.25 / (0.03 \times 85)} = 2.6 m/s$$