[1]

$$
\begin{aligned}
v &= \frac{Q}{\pi R^2} = \frac{1}{\pi R^2}\int_0^R 2\pi(R-y)udr \\
&= \frac{1}{\pi R^2}\int_R^0 2\pi(R-y)udy; \quad u = U + \frac{u^*}{\kappa}ln\frac{y}{R} \\
&= \frac{2\pi}{\pi R^2}\int_R^0 (U + \frac{u^*}{\kappa}ln\frac{y}{R}(R-y)dy \\
&= 2\int_1^0 (U + \frac{u^*}{\kappa}ln\frac{y}{R})(1-\frac{y}{R})d(\frac{y}{R}); \quad \frac{y}{R} = \eta \\
&= 2\int_1^0 (U + \frac{u^*}{\kappa}ln\eta)(1-\eta)d\eta \\
&= 2\int_1^0 (U + \frac{u^*}{\kappa}ln\eta - U\eta - \frac{u^*}{\kappa}\eta ln\eta)d\eta \\
&= 2[U\eta + \frac{u^*}{\kappa}(\eta ln\eta - \eta) - \frac{U}{2}\eta^2 - \frac{u^*}{\kappa}(ln\eta\frac{\eta^2}{2} - \frac{\eta^2}{4})]_1^0 \\
&= 2[U - \frac{U}{2} - \frac{u^*}{\kappa} + \frac{u^*}{\kappa}\frac{1}{4}] \\
&= U - \frac{3}{2}\frac{u^*}{\kappa} = U - 3.75u^*(\kappa = 0.4) \\
&(\int lnxdx = xlnx - x; \quad \int xlnxdx = \int lnxd(\frac{x^2}{2})dx = lnx\frac{x^2}{2} - \int\frac{1}{x}\frac{x^2}{2}dx = lnx\frac{x^2}{2} - \frac{x^2}{4}
\end{aligned}
$$

[2]

$$
\begin{aligned}
&\frac{\tau}{\tau_w} = \frac{r}{R} = (1-\frac{y}{R}) \\
&\tau = \rho l^2(\frac{du}{dy})^2 = \tau_w(1-\frac{y}{R}) \\
&l = \frac{u^*\sqrt{1-y/R}}{du/dy} \\
&\frac{u}{U} = (\frac{y}{R})^{1/7}; \quad \frac{du}{dy} = \frac{U}{R}\frac{1}{7}(\frac{y}{R})^{6/7} \\
&\frac{l}{R} = \frac{u^*}{U}7(\frac{y}{R})^{6/7}\sqrt{1-y/R}
\end{aligned}
$$