

# 完全流体力学 試験問題

1995-9-21, 12:50~14:20

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1. (20) 速度成分が  $u = ax + by$ ,  $v = cx + dy$  で示される流れが非圧縮性流体となるための条件を示せ. また, 流れが渦なし流れとした場合の流れ関数を求めよ. 2. (30) 複素ポテンシャルが次式で表される流れの型を説明し, かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ.

$$(1) w = aze^{i\alpha} \ (\alpha > 0), \quad (2) w = z^n \ (n = \frac{1}{2}), \quad (3) w = -5i \ln z + 3z, \quad (4) w = 2z + 3 \ln z$$

3. (20) 速度  $U$  の一様流れ中に強さ  $Q$  の吹き出しが原点にある場合, この流れ場に作用する力を求めよ.

4. (20) (1) 二次元の渦流れにおいて, 速度成分が  $u = 4y$ ,  $v = 2x$  なる流れは理論上存在しうるか. (2) その流れの流線を求めよ. (3) 直線  $y = 1$ ,  $y = 3$ ,  $x = 2$ ,  $x = 5$  で区切られた長方形のまわりの循環値を求めよ.

5. (20) 図に示すような流線図より, この流れはどのような型の流れを組み合わせたものかを説明せよ. また数値も含めた複素ポテンシャルを求めよ.

(解)

1.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0$$

$$u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy$$

$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$

$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$

$$\text{For irrotational flow, } \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \quad b = c, \quad \psi = axy + \frac{b}{2}(y^2 - x^2) + const.$$

2.

(1) Parallel flow with  $\theta = \alpha$

$$w = ar\{\cos(\theta + \alpha) + i \sin(\theta + \alpha)\}$$

$$\varphi = ar \cos(\theta + \alpha), \quad \psi = ar \sin(\theta + \alpha)$$

$$\frac{dw}{dz} = ae^{i\alpha} = a(\cos \alpha + i \sin \alpha) = u - iv$$

$$u = a \cos \alpha, \quad v = -a \sin \alpha, \quad V = a$$

(2) Corner flow with  $\theta = 2\pi$

$$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i \sin n\theta)$$

$$\varphi = r^n \cos n\theta, \quad \psi = r^n \sin n\theta$$

$$\text{For } n = \frac{1}{2}, \quad \varphi = r^{1/2} \cos \frac{\theta}{2}, \quad \psi = r^{1/2} \sin \frac{\theta}{2}$$

(3) Parallel ( $U=3$ )+circulation( $\Gamma = 10\pi$ ) flow

$$w = -5i \ln(re^{i\theta}) + 3re^{i\theta} = -5 \ln r + 5\theta + 3r(\cos \theta + i \sin \theta)$$

$$\varphi = 5\theta + 3r \cos \theta, \quad \psi = 3r \sin \theta - 5 \ln r$$

(4) Parallel flow( $U=2$ )+source flow( $Q = 6\pi$ )

$$w = 2re^{i\theta} + 3 \ln(re^{i\theta})$$

$$\varphi = 2r \cos \theta + 3 \ln r, \quad \psi = 2r \sin \theta + 3\theta$$

3.

$$w = Uz + m \ln z, \quad m = \frac{Q}{2\pi}$$

$$\frac{dw}{dz} = U + \frac{m}{z}$$

$$\left(\frac{dw}{dz}\right)^2 = U^2 + \frac{m^2}{z^2} + \frac{2Um}{z}$$

$$F_x - iF_y = \frac{i\rho}{2} \oint \left(\frac{dw}{dz}\right)^2 dz = \frac{i\rho}{2} 2Um(2\pi i)$$

$$F_x = -\rho UQ, \quad F_y = 0$$

4.

$$(1) \operatorname{div} V = 0$$

$$(2) \frac{dx}{4y} = \frac{dy}{2x}, \quad 2x dx - 4y dy = 0, \quad x^2 - 2y^2 = c$$

$$(3) 4(5-2) + 10(3-1) - 12(5-1) - 4(1-3) = 12m^2/s$$

$$\Gamma = \int_2^5 \int_1^3 \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx dy$$

$$= - \int_1^3 6 dy = -(18-6) = -12m^2/s$$

5.

$$w = iUz + m \ln \frac{z - z_2}{z - z_1}, \quad z_1 = 0, \quad z_2 = 3 + 4i$$

$$U = 4m/s, \quad m = \frac{Q}{2\pi} = \frac{27 \times 1 \times 4}{2\pi} = \frac{54}{\pi}$$

$$w = i4z + \frac{54}{\pi} \ln \left[1 - \frac{3+4i}{z}\right]$$