完全流体力学 試験問題

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1. (25) 図に示すような流線図より, この流れはどういう型の流れを組み合わせたものかを説明せよ. また数値も含めた複素ポテンシャルを求めよ.
2. (25) 速度成分が $u = ax + by,\ v = cx + dy$ で示される流れが非圧縮性流体となるための条件を示せ．また，流れが渦なし流れとした場合の流れ関数を求めよ．
3. (30) 複素ポテンシャルが次式で表される流れの型を説明し、かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ．

$$(1)\ w = aze^{i\alpha}\ (\alpha > 0),\ (2)\ w = z^n\ (n = \frac{1}{2})$$
$$(3)\ w = -5i\ln z + 3z,\ (4)\ w = 2z + 3\ln z$$

4. (20) 二次元の渦流れで, その速度成分が $v_r = 0,\ v_\theta = \omega$ なるときの渦度を求めよ.

（解）

1.

$$w = iUz + m\ln\frac{z - z_2}{z - z_1},\ z_1 = 0,\ z_2 = 3 + 4i$$
$$U = 4m/s,\ m = \frac{Q}{2\pi} = \frac{27 \times 1 \times 4}{2\pi} = \frac{54}{\pi}$$
$$w = i4z + \frac{54}{\pi}\ln[1 - \frac{3 + 4i}{z}]$$

2.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0$$
$$u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy$$
$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$
$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$
$$\text{For irrotational flow, } \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x},\ b = c,\ \psi = axy + \frac{b}{2}(y^2 - x^2) + const.$$

3.

(1) Parallel flow with $\theta = \alpha$

$$w = ar\{(\cos(\theta + \alpha) + i\sin(\theta + \alpha)\}$$
$$\varphi = ar\cos(\theta + \alpha), \quad \psi = ar\sin(\theta + \alpha)$$
$$\frac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i\sin\alpha) = u - iv$$
$$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$$

(2) Corner flow with $\theta = 2\pi$

$$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$
$$\varphi = r^n\cos n\theta, \quad \psi = r^n\sin n\theta$$

$For\ n = \frac{1}{2}, \quad \varphi = r^{1/2}\cos\frac{\theta}{2}, \quad \psi = r^{1/2}\sin\frac{\theta}{2}$

(3) Parallel (U=3)+circulation($\Gamma = 10\pi$) flow

$w = -5i\ln(re^{i\theta}) + 3re^{i\theta} = -5\ln r + 5\theta + 3r(\cos\theta + i\sin\theta)$

$\varphi = 5\theta + 3r\cos\theta, \quad \psi = 3r\sin\theta - 5\ln r$

(4) Parallel flow(U=2)+source flow($Q = 6\pi$)

$w = 2re^{i\theta} + 3\ln(re^{i\theta})$

$\varphi = 2r\cos\theta + 3\ln r, \quad \psi = 2r\sin\theta + 3\theta$

4.

$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 0, \quad \psi = f(r)$

$v_\theta = -\frac{\partial\psi}{\partial r} = \omega, \quad \psi = -\omega r + f(\theta)$

$\psi = -\omega r, \quad r = (x^2 + y^2)^{1/2}$

$\zeta = -\nabla^2\psi = -\frac{\omega}{r}$