完全流体力学 試験問題

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１. 複素ポテンシャルが次式で表される流れについて説明せよ。

$$(1)\ w = aze^{i\alpha}\ (\alpha > 0),\ (2)\ w = z^n\ (n = \frac{1}{2}),\ (3)\ w = -5i\ln z + 3z,\ (4)\ w = 2z + 3\ln z$$

2. 速度成分が $u = x + y, v = x^2 - y$ で表される流れにおいて $x = \pm 2, y = \pm 2$ の直線からなる正方形の回りの循環値を求めよ。

3. 速度成分が $u = ax + by,\ v = cx + dy$ で示される流れが非圧縮性流体となるための条件を示せ．また，流れが渦なし流れとした場合の流れ関数を求めよ．

4. 図（板書）に示すような $4a$ の長さの平板に α なる傾きをもち，かつ循環をもつ流れがある．(1) 流れの複素ポテンシャルを求めよ．(2) 平行流れ (w-平面) から平板に至る写像関係を示し，かつ流れをスケッチせよ．(3) 平板の後端に岐点がくるようにしたときの循環値をを求めよ．

（解）

.1.

(1) Parallel flow with $\theta = \alpha$

$$w = ar\{(\cos(\theta + \alpha) + i\sin(\theta + \alpha)\}$$

$$\varphi = ar\cos(\theta + \alpha), \quad \psi = ar\sin(\theta + \alpha)$$

$$\frac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i\sin\alpha) = u - iv$$

$$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$$

(2) Corner flow with $\theta = 2\pi$

$$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$

$$\varphi = r^n\cos n\theta, \quad \psi = r^n\sin n\theta$$

$$For\ n = \frac{1}{2}, \quad \varphi = r^{1/2}\cos\frac{\theta}{2}, \quad \psi = r^{1/2}\sin\frac{\theta}{2}$$

(3) Parallel (U=3)+circulation($\Gamma = 10\pi$) flow

$$w = -5i\ln(re^{i\theta}) + 3re^{i\theta} = -5\ln r + 5\theta + 3r(\cos\theta + i\sin\theta)$$

$$\varphi = 5\theta + 3r\cos\theta, \quad \psi = 3r\sin\theta - 5\ln r$$

(4) Parallel flow(U=2)+source flow($Q = 6\pi$)

$$w = 2re^{i\theta} + 3\ln(re^{i\theta})$$

$$\varphi = 2r\cos\theta + 3\ln r, \quad \psi = 2r\sin\theta + 3\theta$$

2.

$$\Gamma = \int\int(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})dxdy$$

$$= \int_{-2}^{2}\int_{-2}^{2}(2x - 1)dxdy = \int_{-2}^{2}(x^2 - x)|_{-2}^{2}dy$$

$$= -4y|_{-2}^{2} = -16$$

3.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0$$

$$u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy$$

$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$

$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$

For irrotational flow, $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$, $b = c$, $\psi = axy + \dfrac{b}{2}(y^2 - x^2) + const.$

4.

$$w = U(z_1 + \frac{a^2}{z_1}) - \frac{i\Gamma}{2\pi}\ln z_1, \quad z_2 = z_1 e^{i\alpha}, \quad z = z_2 + \frac{a^2}{z_2}$$

$$\frac{dw}{dz_1}\frac{dz_1}{dz_2}\frac{dz_2}{dz} = 0$$

$$\frac{dw}{dz_1})_A = U(1 - \frac{a^2}{z_1^2}) - \frac{i\Gamma}{2\pi z_1} = 0$$

$$At\ point\ A,\ z = 2a,\ z_2 = a, \quad z_1 = z_2 e^{-i\alpha} = ae^{-i\alpha}$$

$$\frac{dw}{dz_1})_A = U(1 - \frac{a^2}{a^2 e^{-2i\alpha}}) - \frac{i\Gamma}{2\pi a e^{-i\alpha}} = 0$$

$$U(1 - e^{2i\alpha}) - \frac{i\Gamma}{2\pi a}e^{i\alpha} = 0$$

$$U(e^{-i\alpha} - e^{i\alpha}) - \frac{i\Gamma}{2\pi a} = 0$$

$$U(\cos\alpha - i\sin\alpha - \cos\alpha - i\sin\alpha) - \frac{i\Gamma}{2\pi a} = 0$$

$$\Gamma = -4\pi a U \sin\alpha \ (\Gamma :\ negative)$$