理想流体力学試験問題

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1. 次の流れが理論上存在するための z 方向の速度成分を求めよ。

$$(1)u = x^2 + y^2 + z^2, v = -xy - yz - xz, w =$$
$$(2)u = ln(y^2 + az^2), v = sin(x^2 + y^2), w =$$

2. x 軸にある傾きをもつ一様平行流れ中に置かれた任意断面の柱状体に循環があるとき，柱状体に作用する力を求めよ．

3. 速度成分が $u = x + y, v = x^2 - y$ で表される流れにおいて $x = \pm 2, y = \pm 2$ の直線からなる正方形の回りの循環値を求めよ。

4. 複素ポテンシャルが次式で表される流れの型を説明し、かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ．

$$(1)\ w = aze^{i\alpha}\ (\alpha > 0),\ (2)\ w = z^n\ (n = \frac{2}{3})$$

$$(3)\ w = -i\ln z + 5z,\ (4)\ w = 3z + 2\ln z$$

（解）

1 .

$$(1)\frac{\partial u}{\partial x} = 2x, \frac{\partial v}{\partial y} = -x - z, divV = 0$$
$$\frac{\partial w}{\partial z} = -x + z, w = -xz + 1/2z^2 + f(x, y)$$
$$(2)\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 2ycos(x^2 + y^2)$$
$$w = -2yzcos(x^2 + y^2) + f(x, y)$$

2.

$$\frac{dw}{dz} = Ue^{i\alpha} + \frac{A}{z} + \frac{B}{z^2} + \cdots$$
$$w = Ue^{i\alpha}z + a\ln x - \frac{b}{z} + \cdots$$
$$A = -\frac{i\Gamma}{2\pi}$$
$$(\frac{dw}{dz})^2 = U^2e2i\alpha - \frac{i\Gamma Ue^{i\alpha}}{\pi z}\cdots = A_o + \frac{A_1}{z} + \frac{A_2}{z^2} + \cdots$$
$$A_1 = -\frac{i\Gamma Ue^{i\alpha}}{\pi}$$
$$F_x - F_y = -\pi\rho A_1 = \pi\rho\frac{i\Gamma Ue^{i\alpha}}{\pi} = -i\Gamma\rho Ue^{i\alpha}$$
$$F_x = 0, \quad F_y = -\rho U\Gamma\ (\alpha = 0)$$

3.

$$\Gamma = \int\int (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) dxdy$$
$$= \int_{-2}^{2}\int_{-2}^{2} (2x-1)dxdy = \int_{-2}^{2} (x^2 - x)|_{-2}^{2} dy$$
$$= -4y|_{-2}^{2} = -16$$

4.

(1) Parallel flow with $\theta = \alpha$

$w = ar\{(\cos(\theta+\alpha) + i\sin(\theta+\alpha)\}$

$\varphi = ar\cos(\theta+\alpha), \quad \psi = ar\sin(\theta+\alpha)$

$\frac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i\sin\alpha) = u - iv$

$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$

(2) Corner flow with $\theta = \frac{3}{2}\pi$

$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$

$\varphi = r^n\cos n\theta, \quad \psi = r^n \sin n\theta$

$For\ n = \frac{2}{3}, \quad \varphi = r^{2/3}\cos\frac{2\theta}{3}, \quad \psi = r^{2/3}\sin\frac{2\theta}{3}$

(3) Parallel (U=5)+circulation($\Gamma = 2\pi$) flow

$w = -i\ln(re^{i\theta}) + 5re^{i\theta} = -i\ln r + \theta + 5r(\cos\theta + i\sin\theta)$

$\varphi = \theta + 5r\cos\theta, \quad \psi = 5r\sin\theta - \ln r$

(4) Parallel flow(U=3)+source flow($Q = 4\pi$)

$w = 3re^{i\theta} + 2\ln(re^{i\theta})$

$\varphi = 3r\cos\theta + 2\ln r, \quad \psi = 3r\sin\theta + 2\theta$